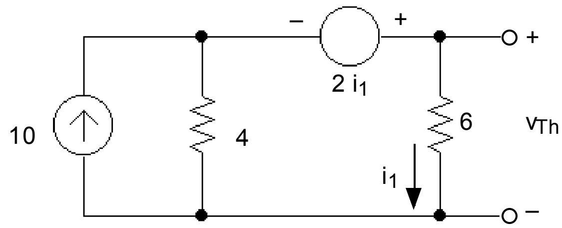
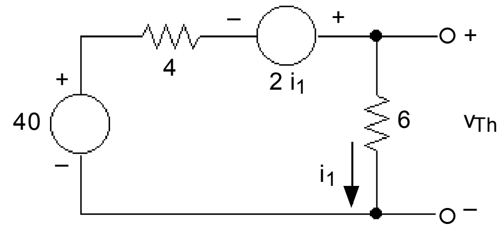
Solución del 1º Parcial

1.- Lo más sencillo es hallar el equivalente de Thevenin para llevar el circuito a la forma RLC serie.

Cálculo de V_{Th} :



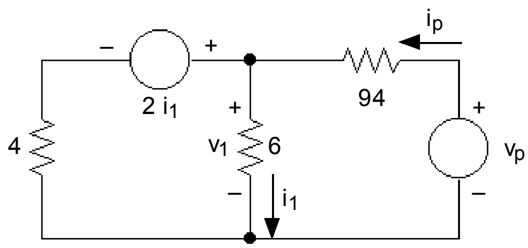


$V_{Th} = 6\, i_1$

$40 + 2\, i_1 = 10\, i_1$

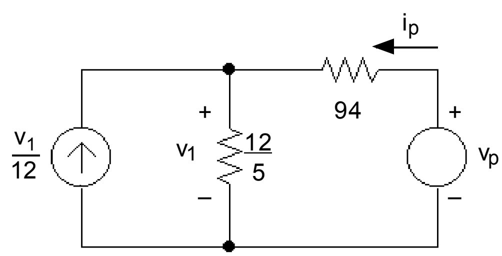
$40 = 8\, i_1 \quad \Rightarrow \quad i_1 = 5$ A. $\quad \Rightarrow \quad V_{Th} = 30$ V.

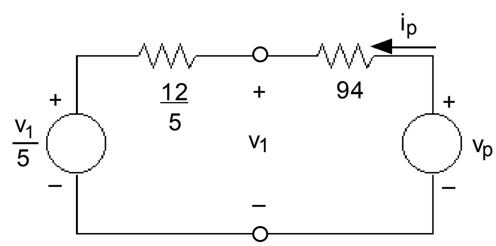
Cálculo de R_{Th} :



$i_1 = \frac{v_1}{6}$

$2\, i_1 = \frac{v_1}{3}$

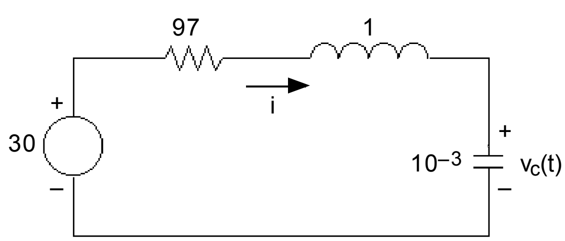




$v_1 = \frac{v_1}{5} + \frac{12}{5} i_p \quad \Rightarrow \quad v_1 = 3\, i_p$

$v_p - \frac{v_1}{5} = \frac{12}{5} i_p + 94\, i_p \quad \Rightarrow \quad v_p = 97\, i_p \quad \Rightarrow \quad R_{Th} = 97\, \Omega$

Sustituyendo el modelo equivalente en t > 0:



$i(t) = C\, v_c'(t) = 10^{-3} v_c'(t)$

$R\, i + L\, i' + v_c = 30$

$0{,}097\, v_c' + 10^{-3} v_c'' + v_c = 30$

$v_c'' + 97\, v_c' + 1000\, v_c = 30.000$

Ec. característica: $\quad s^2 + 97\, s + 1000 = 0$

$s = \frac{-97 \pm \sqrt{9409 - 4000}}{2} \quad \Rightarrow \quad s_1 = -11{,}727$

$s_2 = -85{,}273$

$v_{ch}(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t}$

$v_{cp} = A \quad \Rightarrow \quad 1000A = 30000 \quad \Rightarrow \quad A = 30$

$v_c(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t} + 30$

$v_c'(t) = s_1 K_1 e^{s_1 t} + s_2 K_2 e^{s_2 t}$

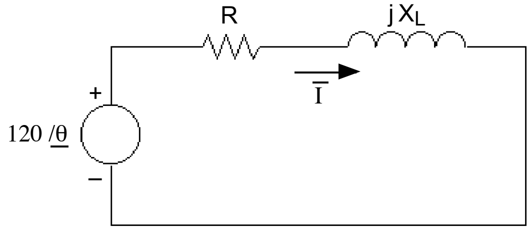
$v_c(0) = K_1 + K_2 + 30 = 10$

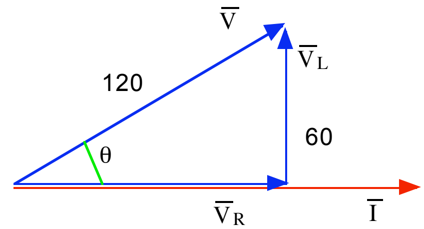
$v_c'(0) = s_1 K_1 + s_2 K_2 = i(0)/C = 0$

$\Rightarrow \begin{cases} K_1 + K_2 = -20 \\ s_1 K_1 + s_2 K_2 = 0 \end{cases} \quad \Rightarrow \quad \begin{matrix} K_1 = -23{,}189 \\ K_2 = 3{,}189 \end{matrix}$

$v_c(t) = -23{,}189\, e^{s_1 t} + 3{,}189\, e^{s_2 t} + 30$ V, $\quad t > 0$

2.- Tomando como referencia a la corriente, podemos dibujar el circuito y su diagrama fasorial (valores rms):



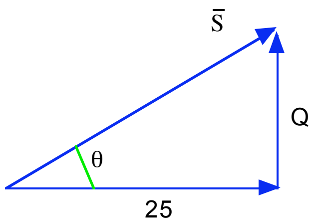


$\Rightarrow \ \text{sen}\,\theta = \dfrac{60}{120}, \qquad \theta = 30^{\circ}$

$\left|\overline{V}_R\right| = 120 \cos\theta = 60\sqrt{3}\ V_{rms}$

La potencia activa es de 25 W, por lo tanto $\left|\overline{V}_R\right| . \left|\overline{I}\right| = 25$ e $\left|\overline{I}\right| = \dfrac{25}{\left|\overline{V}_R\right|} = \dfrac{5\sqrt{3}}{36} A_{rms} = 0{,}2406\ A_{rms}$.

Del triángulo de potencia:



$Q = 25 \tan\theta\ \ VAR = 25\dfrac{\sqrt{3}}{3} VAR$

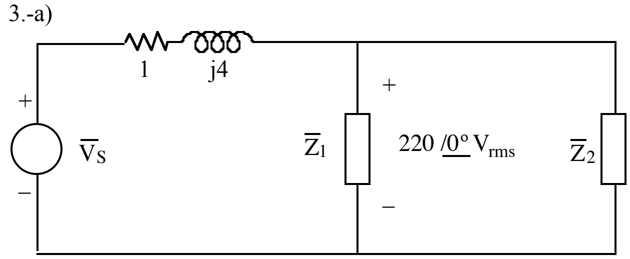
$Q = \left|\overline{I}\right|^2 . X_L \qquad \Rightarrow \qquad X_L = \dfrac{Q}{\left|\overline{I}\right|^2} = 144\sqrt{3}\ \Omega$

Finalmente,

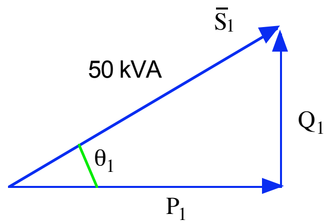
$P = \left|\overline{I}\right|^2 . R \qquad \Rightarrow \qquad \boxed{R = \dfrac{25}{\left|\overline{I}\right|^2} = 432\ \Omega}$

$X_L = 2\pi . f . L \qquad \Rightarrow \qquad \boxed{L = \dfrac{X_L}{2\pi . f} = 793{,}9\ mH}$

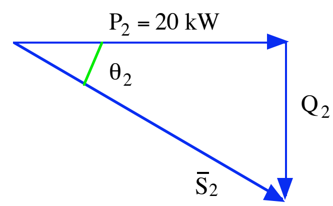
3.-a)



$\overline{Z}_1$:



$\overline{Z}_2$:



$\theta_1 = \arccos(0{,}85) = 31{,}78°$

$\overline{S}_1 = 50(\cos\theta_1 + j\,\mathrm{sen}\,\theta_1)\ \mathrm{kVA} = (42{,}5 + j26{,}84)\ \mathrm{kVA}$

$\theta_2 = -\arccos(0{,}78) = -38{,}74°$

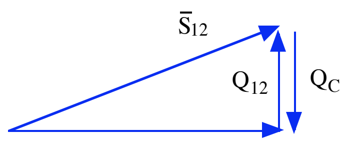
$\overline{S}_2 = 20(1 + j\tan\theta_2)\ \mathrm{kVA} = (20 - j16{,}05)\ \mathrm{kVA}$

$$\overline{S}_{12} = \overline{V}.\overline{I}^* = \overline{S}_1 + \overline{S}_2 = (62{,}5 + j10{,}29)\ \mathrm{kVA}$$

$$\overline{I} = \left(\frac{\overline{S}_{12}}{\overline{V}}\right)^* = \left(\frac{63{,}34\ \mathrm{k}\ \underline{/9{,}35°}}{220\ \underline{/0°}}\right)^* = 287{,}9\ \underline{/-9{,}35°}\ A_{rms}$$

$$\boxed{\overline{V}_S = (1 + j4).\overline{I} + 220\ \underline{/0°} = 691{,}25 + j1089{,}6\ V_{rms}}$$

b) Como la combinación de las dos impedancias tiene una Q positiva (inductiva), se requiere un condensador para reducir el ángulo de la impedancia. Del triángulo de potencia:



$$\overline{S}_C = jQ_C = -jQ_{12}$$

$$\overline{S}_C = \frac{220^2}{\overline{Z}_C^*} = \frac{220^2}{j10{,}29\ \mathrm{k}} = -j\,4{,}702\ \Omega$$

$$2\pi.f.C = \frac{1}{4{,}702} \quad \Rightarrow \quad \boxed{C = \frac{1}{2\pi.60.4{,}702} = 564\ \mu F}$$